E-Technikklausur

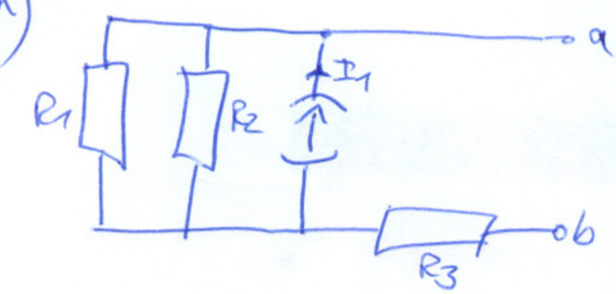
Sommersemester

2005

Klausur SoSe 2005

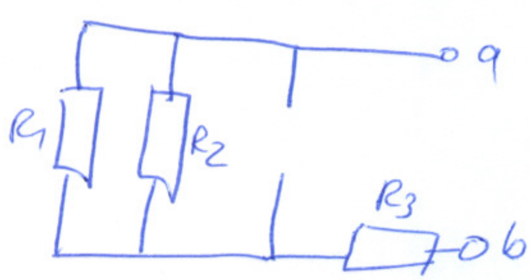
2.2

a)



Bestimmung von R_i:

Ersatzschaltung:



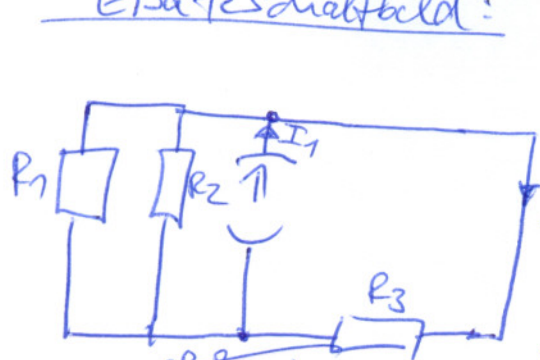
$R_i = ?$

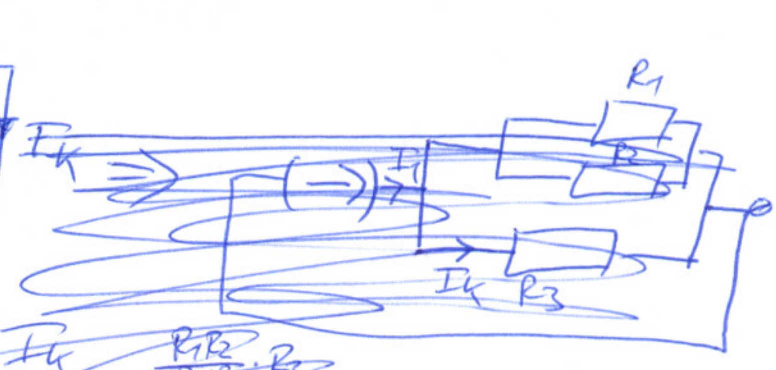
Ersatzschaltung II:

$$R_i = \frac{R_1R_2}{R_1+R_2} + R_3 = \frac{R_1R_2 + R_3(R_1+R_2)}{R_1+R_2} = \frac{R_1R_2+R_1R_3+R_2R_3}{R_1+R_2}$$

Bestimmung von I_k:

Ersatzschaltbild:

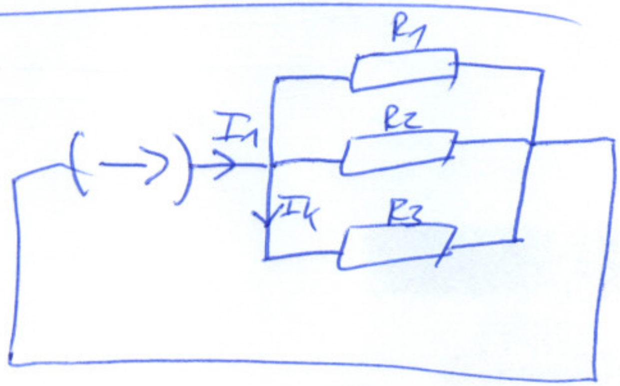




$$\cancel{\frac{I_k}{I_1} = \frac{\left(\frac{R_1R_2}{R_1+R_2}+R_3\right)}{R_3}}$$

$$\cancel{\frac{I_k}{I_1} = \frac{\frac{R_1R_2}{R_1+R_2}\cdot R_3}{R_3}}$$

Ersatzschaltbild II:



Also:

$$\frac{I_k}{I_1} = \frac{\frac{R_1R_2\cancel{R_3}}{R_1R_2+R_1R_3+R_2R_3}}{\cancel{R_3}} = \frac{R_1R_2}{R_1R_2+R_1R_3+R_2R_3} \Rightarrow \cancel{I_k = I_1}$$

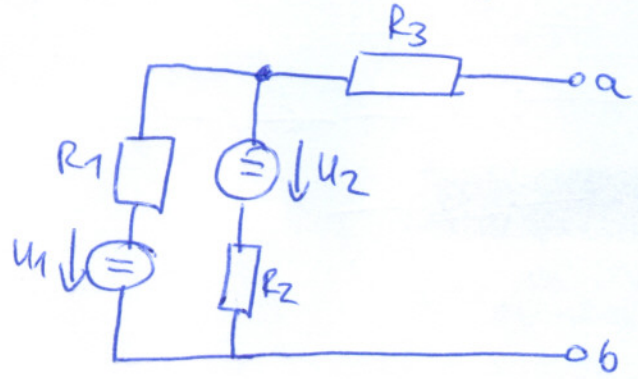
$$\Rightarrow I_k = I_1 \cdot \frac{R_1R_2}{R_1R_2+R_1R_3+R_2R_3}$$

Bestimmung von U_L:

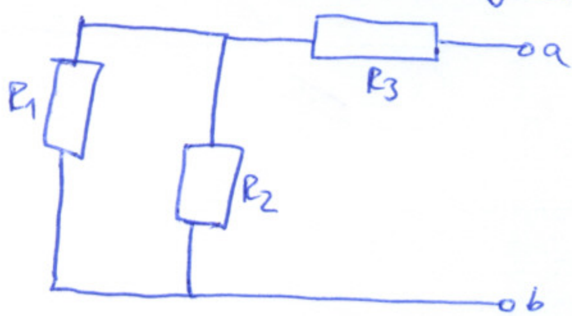
$$\cancel{U_L = R_i \cdot I_k = \left(\frac{R_1R_2}{R_1+R_2} + R_3\right) \cdot \frac{R_1R_2 \cdot I_1}{R_1R_2+R_1R_3+R_2R_3}}$$

$$U_L = R_i \cdot I_k = \frac{\cancel{(R_1R_2+R_1R_3+R_2R_3)}}{R_1+R_2} \cdot \frac{R_1R_2 \cdot I_1}{\cancel{(R_1R_2+R_1R_3+R_2R_3)}} = \frac{R_1R_2}{R_1+R_2} \cdot I_1$$

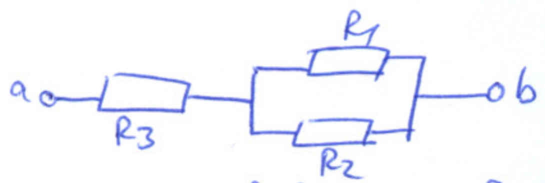
2.2 b) Bestimmung von R_i:



Ersatzschaltung:

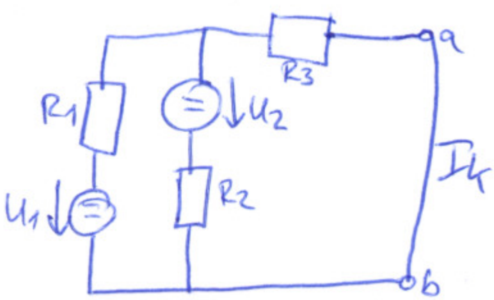


Ersatzschaltung II:



$$R_i = R_3 + \frac{R_1R_2}{R_1+R_2} = \frac{R_1R_2+R_1R_3+R_2R_3}{R_1+R_2} \quad (\text{s. } 2.2a)$$

Bestimmung von I_k:



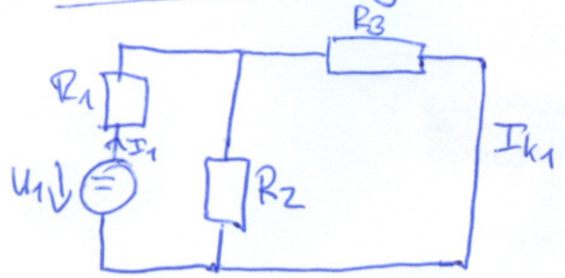
$\Rightarrow$ Superposition

$$I_k = I_{k1} + I_{k2}$$

Bestimmung von I_{k1}:

bei $U_2 = 0$:

Ersatzschaltung



I_{k1} ist der Teilstrom, der durch R_3 geht.
Wir wenden also die Stromteilerregel an.

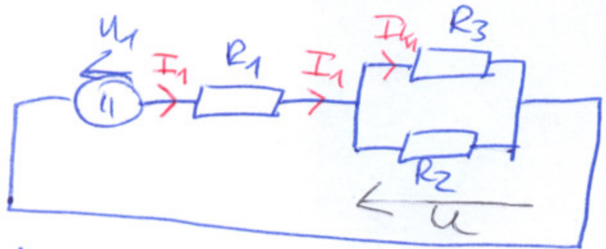
$$\frac{I_{k1}}{I_1} = ?$$

Erste Frage: Was ist I_1? => Berechnen.

$$I_1 = \frac{U_1}{R_1 + \frac{R_2 R_3}{R_2+R_3}} = \underline{\underline{\frac{U_1 \cdot (R_2+R_3)}{R_1R_2+R_1R_3+R_2R_3}}}$$

Jetzt haben wir den Wert für I_1.
I_{k1}, lässt sich leicht erkennen, ist $\frac{U_1-(R_1 \cdot I_1)}{R_3}$.

Machen wir es nicht komplizierter als es ist.
Erstmal die Ersatzschaltung:



Wir können auch statt $I_1 = \frac{U_1}{R_1+\frac{R_2R_3}{R_2+R_3}}$ einfach

$I_1 = \frac{U}{\frac{R_2R_3}{R_2+R_3}} = \frac{U(R_2+R_3)}{R_2R_3}$ rechnen. Dann haben wir wieder das gleiche U_1 um den Kurzschlussstrom zu berechnen:

$$I_{k1} = \frac{U}{R_3}.$$

Also: →

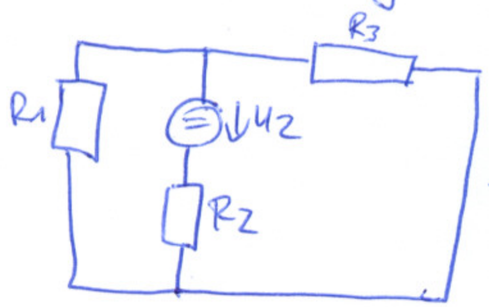
$$\frac{I_{k1}}{I_1} = \frac{\cancel{U}\,\frac{R_2\cancel{R_3}}{R_2+R_3}}{\cancel{U}\,\cancel{R_3}} = \frac{R_2}{R_2+R_3}$$ und somit

$$I_{k1} = I_1 \cdot \frac{R_2}{R_2+R_3}$$

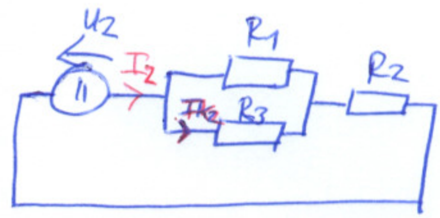
$$I_{k1} = \frac{R_2}{\cancel{R_2+R_3}} \cdot \frac{U_1(\cancel{R_2+R_3})}{R_1R_2+R_2R_3+R_1R_3} = \underline{\underline{\frac{U_1\cdot R_2}{R_1R_2+R_1R_3+R_2R_3}}}$$

Bestimmung von I_{k2} bei $U_1=0$:

~~Ersatzschaltung~~:



I_{k2} $\Longrightarrow$



Auch hier wieder Stromteilerregel:

$$\frac{I_{k2}}{I_2} = ? \quad \text{Was ist } I_2? \Rightarrow I_2 = \frac{U_2}{\frac{R_1R_3}{R_1+R_3}+R_2} = \frac{U_2}{\frac{R_1R_3+R_2(R_1+R_3)}{R_1+R_3}}$$

$$\Rightarrow I_2 = \underline{\frac{U_2(R_1+R_3)}{R_1R_3+R_1R_2+R_2R_3}}$$

$$\frac{I_{k2}}{I_2} = \frac{\cancel{U}\,\frac{R_1\cancel{R_3}}{R_1+R_3}}{\cancel{U}\,\cancel{R_3}} = \underline{\underline{\frac{R_1}{R_1+R_3}}}$$

$$I_{k2} = I_2 \cdot \frac{R_1}{R_1+R_3} = \frac{R_1}{\cancel{R_1+R_3}} \cdot \frac{U_2(\cancel{R_1+R_3})}{R_1R_2+R_1R_3+R_2R_3} = \underline{\underline{\frac{R_1\cdot U_2}{R_1R_2+R_1R_3+R_2R_3}}}$$

$$\boxed{I_k = I_{k1}+I_{k2} = \frac{U_1R_2+U_2R_1}{R_1R_2+R_1R_3+R_2R_3}}$$

Bestimmung von U_L:

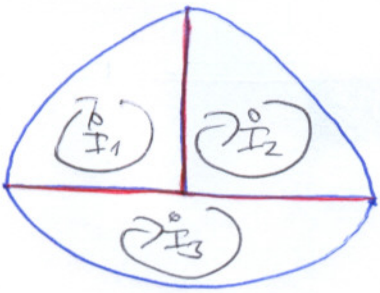
$$U_L = R_i \cdot I_k$$

$$U_L = \frac{\cancel{(R_1R_2 + R_1R_3 + R_2R_3)}}{R_1 + R_2} \cdot \frac{U_1R_2 + U_2R_1}{\cancel{(R_1R_2 + R_1R_3 + R_2R_3)}} = \frac{U_1R_2 + U_2R_1}{R_1 + R_2}$$

Aufgabe 3

a) Graph:

— = Baum



m = 3 Maschen

b)

$$M1: \; R_1 \mathring{I}_1 + R_3 \mathring{I}_1 - R_3 \mathring{I}_2 = -U_1 - U_3$$

$$M2: \; R_3 \mathring{I}_2 - R_3 \mathring{I}_1 + R_2 \mathring{I}_2 + R_6 \mathring{I}_2 - R_6 \mathring{I}_3 = U_2 - U_3$$

$$M3: \; R_4 \cdot \mathring{I}_3 + R_5 \mathring{I}_3 + R_6 \mathring{I}_3 - R_6 \mathring{I}_2 = 0$$

Matrixdarstellung

$$\begin{pmatrix} R_1 + R_3 & -R_3 & 0 \\ -R_3 & R_2 + R_6 + R_3 & -R_6 \\ 0 & -R_6 & R_4 + R_5 + R_6 \end{pmatrix} \begin{pmatrix} \mathring{I}_1 \\ \mathring{I}_2 \\ \mathring{I}_3 \end{pmatrix} = \begin{pmatrix} -U_1 - U_3 \\ U_2 + U_3 \\ 0 \end{pmatrix}$$

c)

$$I_1 = \underline{\underline{\mathring{I}_2 - \mathring{I}_3}} \qquad \underline{\underline{U_{R_1} = R_1 \cdot \mathring{I}_1}}$$

$$I_2 = \underline{\underline{\mathring{I}_2 - \mathring{I}_1}}$$

$$I_3 = \underline{\underline{\mathring{I}_3}}$$

d)

$$I_1 = \mathring{I}_2 - \mathring{I}_3 = \mathring{I}_2 - \frac{2U}{13R}$$

Masche 3: $-R \cdot \mathring{I}_2 + 3R \cdot \mathring{I}_3 = 0 \quad | -3R\mathring{I}_3$

$$-R\mathring{I}_2 = -3R\mathring{I}_3 \qquad | :(-R)$$

$$\mathring{I}_2 = \frac{3R\mathring{I}_3}{R} = \frac{3R}{R} \cdot \frac{2U}{13R} = \underline{\underline{\frac{6U}{13R}}}$$

Somit:

$$I_1 = \frac{64}{13R} - \frac{24}{13R} = \underline{\underline{\frac{44}{13R}}}$$

Aufgabe 4:

a) $\underline{H}_{(j\omega)} = \dfrac{R_2}{R_1 + \frac{1}{j\omega C} + R_2} = \underline{\underline{\dfrac{j\omega R_2 C}{1 + j\omega C(R_1+R_2)}}}$ | $\underline{\underline{|\underline{H}_{(j\omega)}| = \dfrac{\omega R_2 C}{\sqrt{1+\omega^2 C^2 (R_1+R_2)^2}}}}$

$$\varphi = \varphi_1 - \varphi_2 = \arctan\left(\frac{\omega R_2 C}{0}\right) - \arctan\left(\frac{\omega C(R_1+R_2)}{1}\right)$$

$$= \underline{\underline{\frac{\pi}{2} - \arctan(\omega C(R_1+R_2))}}$$

b)

$\omega = 0$:

$$|\underline{H}_{(j\omega)}| = \frac{0 \cdot R_2 \cdot C}{\sqrt{1+\omega^2 C^2 (R_1+R_2)^2}} = \underline{\underline{0}}$$

$$\varphi_{(j\omega)} = \frac{\pi}{2} - \arctan(0) = \frac{\pi}{2} - 0 = \underline{\underline{\frac{\pi}{2}}}$$

$$\lim_{\omega\to\infty} |\underline{H}_{(j\omega)}| = \frac{\overbrace{\infty R_2 C}^{\to\infty}}{\underbrace{\sqrt{1+\infty^2 C^2 (R_1+R_2)^2}}_{\to\infty}} \Rightarrow \text{l'Hôspital} \ldots$$

$$= \left(\frac{\omega^2 R_2^2 C^2}{1+\omega^2 C^2 (R_1+R_2)^2}\right)^{\frac{1}{2}} \overset{\text{l'Hôspital}}{=} \left(\lim_{\omega\to\infty} \frac{2\omega R_2^2 C^2}{2\omega R_1^2 C^2 + 4\omega C^2 R_1 R_2 + 2\omega C^2 R_2^2}\right)^{\frac{1}{2}}$$

$$= \left(\lim_{\omega\to\infty} \frac{\cancel{2\omega} R_2^2 \cancel{C^2}}{\cancel{2\omega C^2}(R_1^2 + 2R_1R_2 + R_2^2)}\right)^{\frac{1}{2}} = \left(\frac{R_2^2}{R_1^2 + 2R_1R_2 + R_2^2}\right)^{\frac{1}{2}} = \left(\frac{R_2^2}{(R_1+R_2)^2}\right)^{\frac{1}{2}}$$

$$= \underline{\underline{\frac{R_2}{R_1+R_2}}}$$

$$\varphi_{(j\omega)} = \frac{\pi}{2} - \arctan(\infty) = \frac{\pi}{2} - \frac{\pi}{2} = \underline{\underline{0}}$$

$\omega_g = ?$

$$R_1 + R_2 = \frac{1}{\omega_g C} \quad \Big| \cdot \omega_g$$

$$\omega_g (R_1 + R_2) = \frac{1}{C} \quad \Big| : (R_1 + R_2)$$

$$\underline{\underline{\omega_g = \frac{1}{(R_1+R_2)C}}}$$

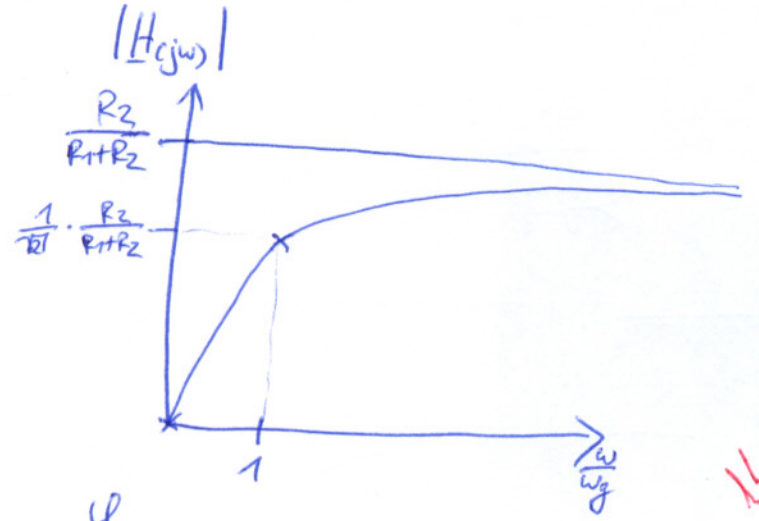
$\omega = \omega_g$:

$$|\underline{H}_{(j\omega)}| = \frac{\frac{1}{(R_1+R_2)\cancel{C}} \cdot R_2 \cancel{C}}{\sqrt{1 + \left(\frac{1}{(R_1+R_2)C}\right)^2 \cdot C^2 \cdot (R_1+R_2)^2}} = \frac{\frac{R_2}{(R_1+R_2)}}{\sqrt{1 + \left(\frac{C(R_1+R_2)}{C(R_1+R_2)}\right)^2}} = \frac{\frac{R_2}{R_1+R_2}}{\sqrt{2}}$$

$$\underline{\underline{= \frac{R_2}{R_1+R_2} \cdot \frac{1}{\sqrt{2}}}}$$

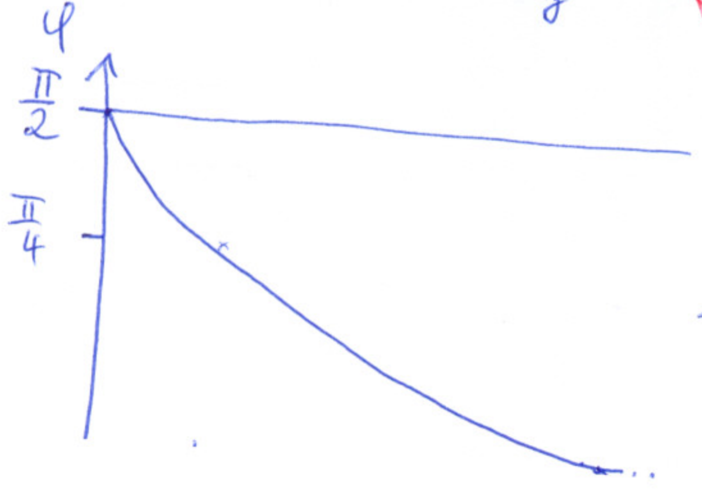
$$\varphi_{(\omega_g)} = \frac{\pi}{2} - \arctan\left(\frac{C(R_1+R_2)}{(R_1+R_2)C}\right) = \frac{\pi}{2} - \arctan(1) = \frac{\pi}{2} - \frac{\pi}{4} = \underline{\underline{\frac{\pi}{4}}}$$

Skizzen:



Betrag

$\Longrightarrow$ Hochpass

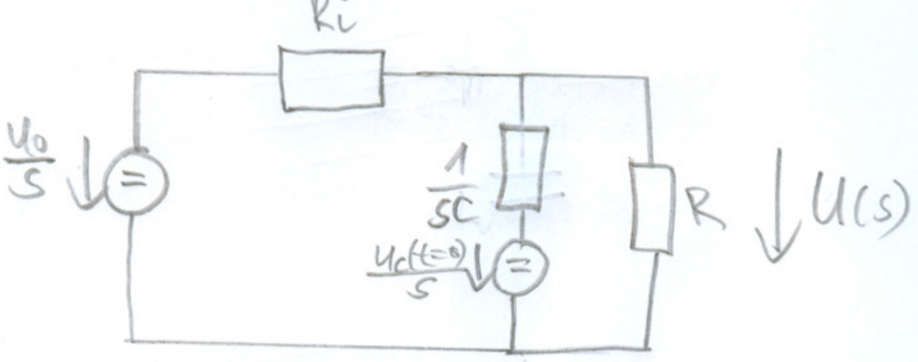


Phase

Aufgabe 5:

$t \geq 0$

a)



$$\cancel{\text{b)}\ \frac{U(s)}{\frac{U_0}{s}} = \frac{\frac{\frac{1}{sC}\cdot R}{\frac{1}{sC}+R}\cdot sC}{R_i + \frac{\frac{1}{sC}\cdot R}{\frac{1}{sC}+R}\cdot sC} = \frac{\frac{R}{1+sCR}\cdot(1+sCR)}{\frac{R}{1+sCR}+sCR_i\cdot(1+sCR)} = \frac{R}{R+sCR_i(1+sCR)}}$$

$$\cancel{= \frac{R}{R+sCR_i+s^2C^2R\cdot R_i}}$$

b)
$$\frac{U(s)}{\frac{U_0}{s}} = \frac{\frac{\frac{1}{sC}\cdot R}{\frac{1}{sC}+R}}{R_i+\frac{\frac{1}{sC}\cdot R}{\frac{1}{sC}+R}} = \frac{\frac{\frac{1}{sC}\cdot R}{\cancel{\left(\frac{1}{sC}+R\right)}}}{\frac{R_i\left(\frac{1}{sC}+R\right)+\frac{1}{sC}\cdot R}{\cancel{\left(\frac{1}{sC}+R\right)}}} = \frac{\frac{R}{sC}}{\frac{R_i}{sC}+R_iR+\frac{R}{sC}} \begin{matrix}\cdot sC\\ \cdot sC\end{matrix}$$

$$= \frac{R}{R_i+R_iR\cdot sC+R} = \frac{R}{sCR_iR+R+R_i} \quad : CR_iR \quad = \frac{\frac{\cancel{R}}{CR_i\cancel{R}}}{s+\frac{R+R_i}{CR_iR}} = \frac{\frac{1}{CR_i}}{s+\frac{R+R_i}{CR_iR}}$$

$$= R\cdot\frac{\frac{1}{CR_i\cdot R}}{s+\frac{R+R_i}{CR_iR}} = \frac{R}{R_i+R}\cdot\frac{\frac{R+R_i}{CR_iR}}{s+\frac{R+R_i}{CR_iR}} \quad \frac{R+R_i}{CR_iR} \to a$$

$$\Rightarrow U(s) = \frac{U_0}{s}\cdot\frac{R}{R_i+R}\cdot\frac{a}{s+a} = \underline{\underline{\frac{U_0\cdot R}{R_i+R}\cdot\frac{a}{s(s+a)}}}$$

c) $u(t) = ?$

$$\frac{a}{s(s+a)} \circ\!\!-\!\!\bullet\; 1 - e^{-at}$$

Also:

$$u(t) = \frac{U_0 \cdot R}{R_i + R} \cdot \left(1 - e^{-at}\right)$$

$$= \underline{\underline{\frac{U_0 \cdot R}{R_i + R} \cdot \left(1 - e^{-\frac{R+R_i}{CR_iR} t}\right)}}$$

d) ~~$U_0 = 15V$; $R_i = 8\,\Omega$~~

Herrn Bauer fragen...