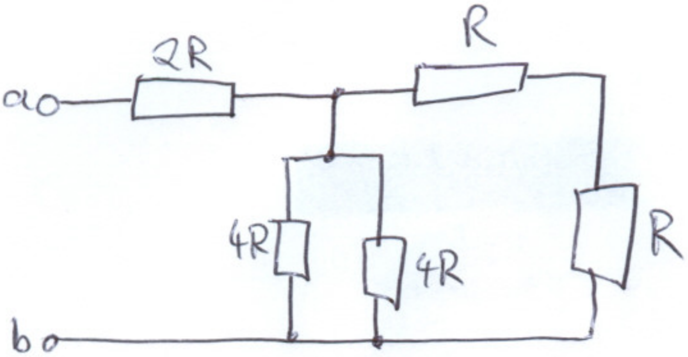
E-Technikklausur

Wintersemester 2007/08

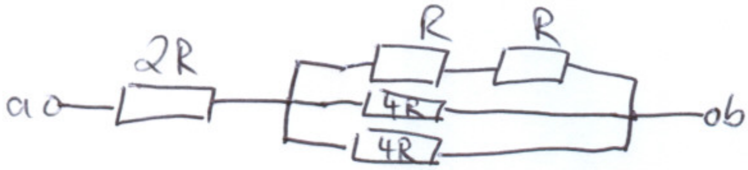
1

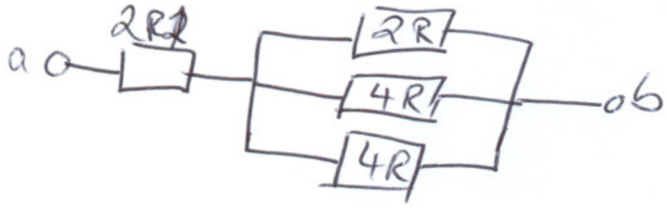
1.1



Gesamtwiderstand bestimmen!

Ersatzschaltung:

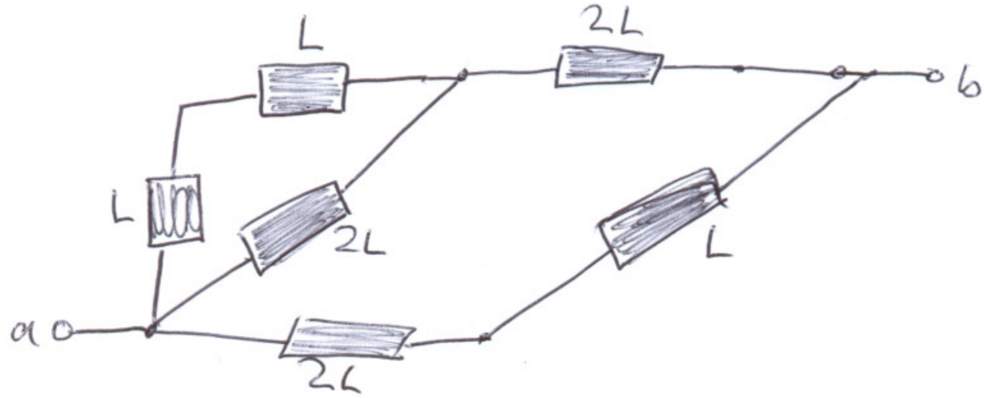




$$R_{ges} = 2R + \frac{2R \cdot 4R \cdot 4R}{2R \cdot 4R + 2R 4R + 4R \cdot 4R} = 2R + \frac{32 R^3}{32 R^2} = 2R + 1R$$

$$= \underline{\underline{3R}}$$

1.2



Gesamtinduktivität bestimmen!

Reihenschaltung bei Spulen:

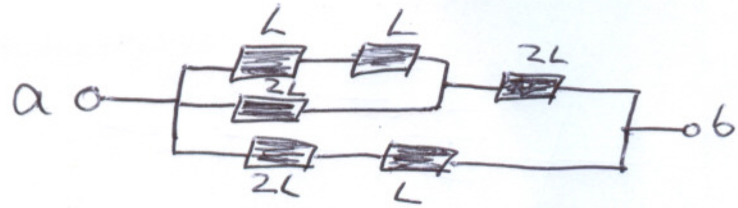
$$L_{ges} = \sum_{\nu=1}^{m} L_\nu$$ (Skript S. 80)

Parallelschaltung bei Spulen:

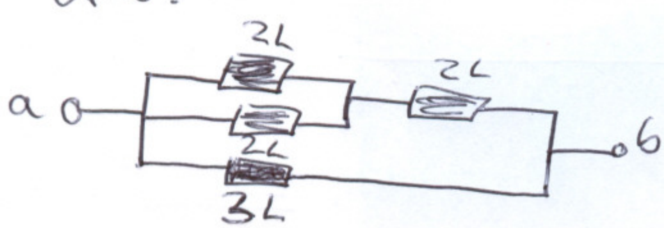
$$\frac{1}{L_{ges}} = \sum_{\nu=1}^{m} \frac{1}{L_\nu}$$ (Skript S. 79)

Hier also:

$L_{ges} = ?$ $\Rightarrow$ Ersatzschaltbild!



Also:



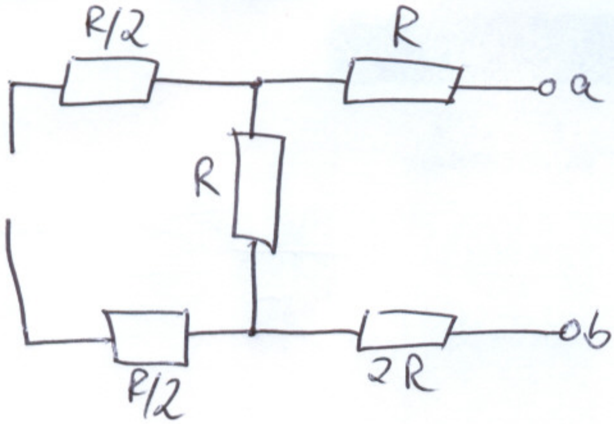
Somit:

$$L_{ges} = \frac{\left(\frac{2L\cdot 2L}{2L+2L} + 2L\right)3L}{\frac{2L\cdot 2L}{2L+2L} + 2L + 3L} = \frac{\left(\frac{4L^{\cancel{2}}}{\cancel{4L}} + 2L\right)\cdot 3L}{\frac{\cancel{4L^2}}{\cancel{4L}} + 2L + 3L} = \frac{9L^2}{6L} = \underline{\underline{\frac{3}{2}L}}$$

$\boxed{1.3}$ ges.: R_i, I_k, U_L

Bestimmung von R_i:

ESB(1):



ESB(2):

~~$R_{ges} = 4R$~~ $R_i = 4R$

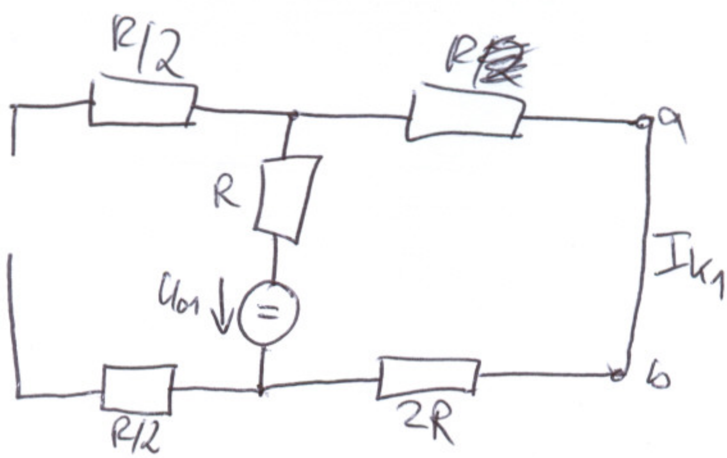
Bestimmung von I_k:

Zwei Quellen, also Superpositionsprinzip!

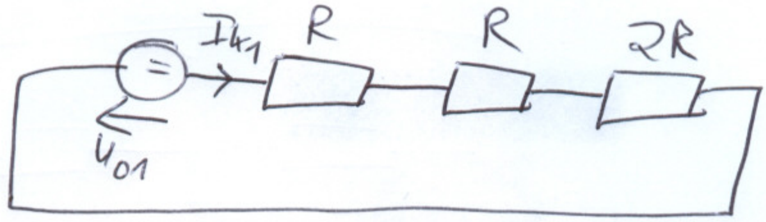
$I_k = I_{k1} + I_{k2}$

I_{k1} bei $I_{01} = 0$:

ESB(1):



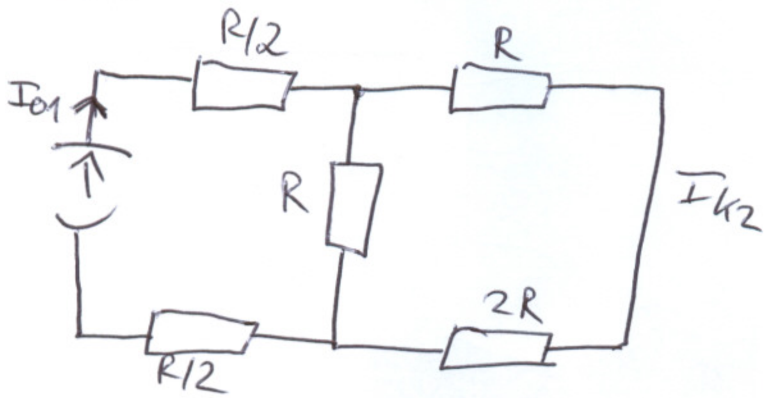
ESB(2):



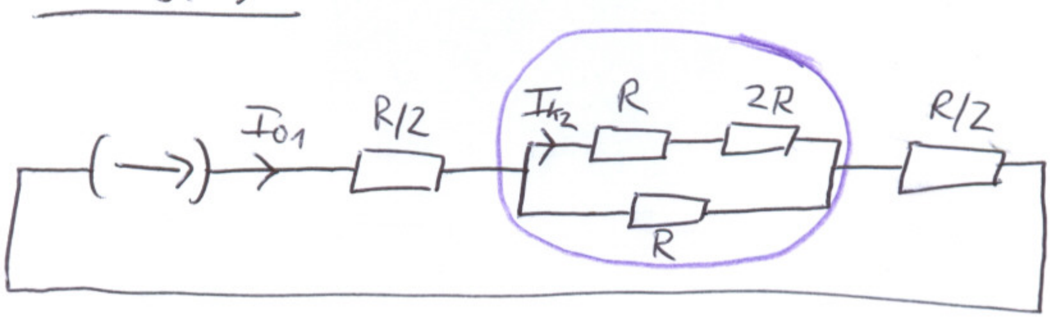
$$I_{k1} = \frac{U_{01}}{R_i} = \underline{\underline{\frac{U_{01}}{4R}}}$$

I_{k2} bei $U_{01} = 0$:

ESB(1):



ESB(2):



$$\frac{I_{k2}}{I_{01}} = \frac{\frac{(R+2R)R}{R+2R+R}}{R+2R} = \frac{\frac{\cancel{3}R^{\cancel{2}}}{4R}}{\cancel{3R}} = \underline{\frac{1}{4}} \Rightarrow \underline{\underline{I_{k2} = \frac{1}{4} I_{01}}}$$

$$I_k = I_{k1} + I_{k2} = \underline{\underline{\frac{U_{01}}{4R} + \frac{1}{4} I_{01}}}$$

$$U_L = R_i \cdot I_k = 4R \cdot \left(\frac{U_{01}}{4R} + \frac{I_{01}}{4}\right) = \boxed{U_{01} + R \cdot I_{01}}$$

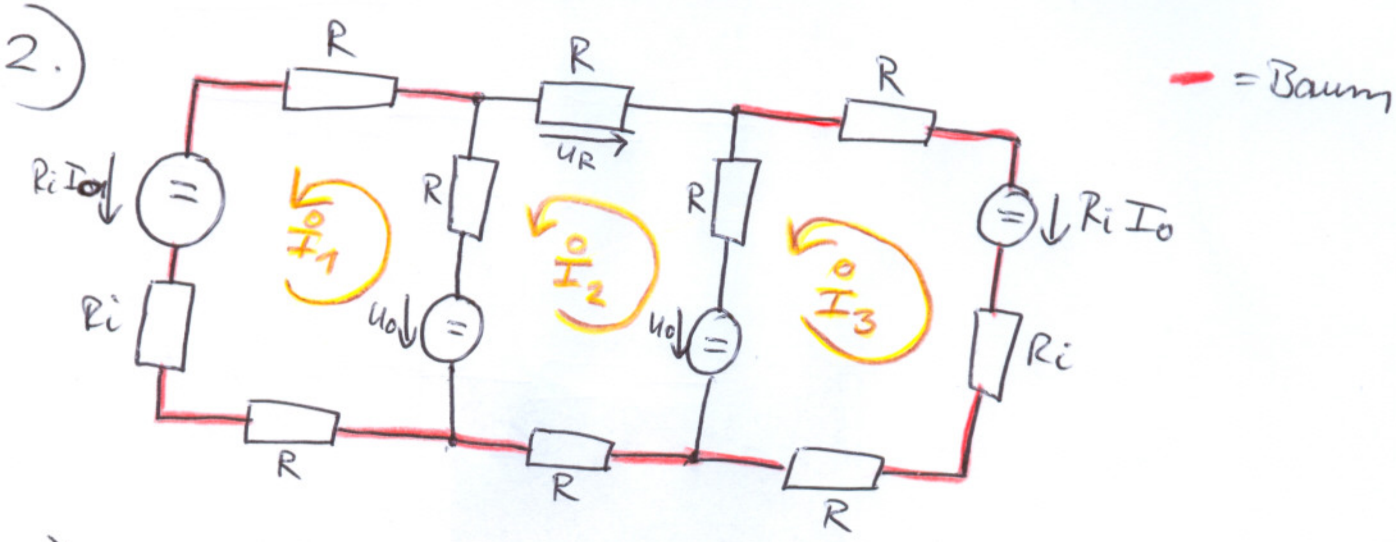
Aufgabe 2

a)

1.) Die Regeln zum Umwandeln von Strom- in Spannungsquelle sind im Skript auf Seite 61 zu sehen...

Dem entsprechend wurde verfahren (~~s. Lösung~~ ~~s. A2~~ (S. 2)).

2.)



3.) $k = 4$ Knoten
$k-1 = 3$ Baumzweige
$z = 6$ Zweige (Graphenzweige)
$m = z - k + 1 = 6 - 4 + 1 = 3$ unabhängige Maschen.

(Hilfe: Formelblatt)

4.) siehe Zeichnung in 2.)

b) Maschenströme:

$$M1: \; R_i \mathring{I}_1 + R\mathring{I}_1 + R\mathring{I}_1 - R\mathring{I}_2 + R\mathring{I}_1 = U_0 - R_i I_0$$

$$M2: \; 4R\mathring{I}_2 - R\mathring{I}_1 - R\mathring{I}_3 = \overbrace{U_0 - U_0}^{=0}$$

$$M3: \; 3R\mathring{I}_3 + R_i\mathring{I}_3 - R\mathring{I}_2 = R_i I_0 - U_0$$

Matrixdarstellung:

$$\boxed{\begin{pmatrix} 3R+R_i & -R & 0 \\ -R & 4R & -R \\ 0 & -R & 3R+R_i \end{pmatrix} \begin{pmatrix} \mathring{I}_1 \\ \mathring{I}_2 \\ \mathring{I}_3 \end{pmatrix} = \begin{pmatrix} U_0 - R_i I_0 \\ 0 \\ R_i I_0 - U_0 \end{pmatrix}}$$

$R_i = R = 10\Omega, \; U_0 = 2V, \; I_0 = 100mA$

konkrete Matrixdarstellung:

$$\begin{pmatrix} 40 & -10 & 0 \\ -10 & 40 & -10 \\ 0 & -10 & 40 \end{pmatrix} \begin{pmatrix} \mathring{I}_1 \\ \mathring{I}_2 \\ \mathring{I}_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

$$\det(A) = 40 \times 40 \times 40 - 4000 - 4000 = 64.000 - 8.000 = \underline{\underline{56.000}}$$

$$\det(A_1) = 40 \times 40 - 100 - 100 = 1.600 - 200 = \underline{\underline{1.400}}$$

$$\det(A_2) = -400 + 400 = \underline{\underline{0}}$$

$$\det(A_3) = 40 \times 40 \times (-1) + 100 + 100 = -1.600 + 200 = \underline{\underline{-1.400}}$$

$$\mathring{I}_1 = \frac{\det(A_1)}{\det(A)} = \frac{1.400}{56.000} = 0{,}025\,A = \underline{\underline{25\,mA}}$$

$$1.400 : 56.000 = \underline{0{,}025}\ A$$

$$\begin{array}{r}
1.400 \\
\underline{0\,000} \\
14000 \\
\underline{00} \\
140.000 \\
\underline{112.000} \\
28\,000 \\
\underline{28\,0.000} \\
\underline{\underline{0}}
\end{array}$$

$$\mathring{I}_2 = \frac{\det(A_2)}{\det(A)} = \frac{0}{56.000} = \underline{\underline{0\,A}}$$

$$\mathring{I}_3 = \frac{\det(A_3)}{\det(A)} = -\frac{1.400}{56.000}$$ ⇒ wie $\mathring{I}_1$, nur umgekehrtes Vorzeichen, also $\underline{\underline{-25\,mA}}$.

d) $$U_R = R \cdot \mathring{I}_2 = 10\,\Omega \cdot 0\,A = \underline{\underline{0\,V}}$$

Aufgabe 3

a) $H_{(j\omega)} = \dfrac{\frac{1}{j\omega C}}{R + j\omega L + \frac{1}{j\omega C}} = \dfrac{1}{1 + j\omega CR - \omega^2 LC} = \underline{\underline{\dfrac{1}{1 - \omega^2 LC + j\omega CR}}}$

b) $|H_{(j\omega)}| = \underline{\underline{\dfrac{1}{\sqrt{(1-\omega^2 LC)^2 + \omega^2 C^2 R^2}}}}$

$\varphi = \arctan(0) - \arctan\left(\dfrac{\omega CR}{1-\omega^2 LC}\right) = \underline{\underline{-\arctan\left(\dfrac{\omega CR}{1-\omega^2 LC}\right)}}$

c) Hier die Rechnung:

$$\frac{1}{\sqrt{(1-\omega_0^2 LC)^2 + (\omega_0 CR)^2}}_{\;1} = \frac{1}{\sqrt{2}}_{\;2}$$

$\Rightarrow$ das heißt, dass $1 = 2$ ergibt.

Also (nach Quadrieren auf beiden Seiten):

$$\underbrace{(1-\omega_0^2 LC)^2}_{\text{~~binomische Formel~~}} + (\omega_0 CR)^2 = 2$$

$\Rightarrow$ Und jetzt rechnen wir schrittweise so weit, bis ω_0 alleine steht.

Für LC setzen wir jetzt $\boxed{\frac{1}{2}R^2C^2}$ ein...

$$\underbrace{\left(1-\omega_0^2 \cdot \boxed{\tfrac{1}{2}R^2C^2}\right)^2}_{\text{binomische Formel}} + (\omega_0 CR)^2 = 2$$

$$\underline{1 - \cancel{2}\cdot\omega_0^2\cdot\tfrac{1}{\cancel{2}}R^2C^2 + \omega_0^4\tfrac{1}{4}R^4C^4} + \omega_0^2 C^2 R^2 = 2$$

$\Rightarrow$ übersichtlicher hinschreiben ergibt ...

$$\Rightarrow \quad 1 \boxed{\cancel{-\omega_0^2 R^2 C^2}} + \frac{\omega_0^4 R^4 C^4}{4} \boxed{\cancel{+\omega_0^2 C^2 R^2}} = 2$$

$$\Rightarrow \quad 1 + \frac{\omega_0^4 R^4 C^4}{4} = 2 \qquad |-1$$

$$\Rightarrow \quad \frac{\omega_0^4 R^4 C^4}{4} = 1 \qquad |\cdot 4$$

$$\Rightarrow \omega_0^4 R^4 C^4 = 4 \quad | \sqrt{\ }$$

$$\Rightarrow \omega_0^2 R^2 C^2 = 2 \quad | \sqrt{\ }$$

$$\Rightarrow \omega_0 RC = \sqrt{2} \quad | : RC$$

$$\Rightarrow \underline{\underline{\omega_0 = \frac{\sqrt{2}}{RC}}}$$

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d)

$\underline{\omega = 0:}$

$$|\underline{H}_{(j\omega)}| = \frac{1}{\sqrt{(1-0^2LC)^2+(0\cdot RC)^2}} = \frac{1}{\sqrt{1}} = \underline{\underline{1}}$$

$$\varphi = -\arctan\left(\frac{0\cdot RC}{1-0^2LC}\right) = -0 = \underline{\underline{0}}$$

$\underline{\omega \to \infty:}$

$$|\underline{H}_{(j\omega)}| = \frac{1}{\sqrt{(1-\infty^2LC)^2+(\infty RC)^2}} = \frac{1}{\infty} = \underline{\underline{0}}$$

$$\varphi = -\arctan\left(\frac{\overset{\to\infty}{\infty RC}}{\underset{\to -\infty}{1-\infty^2 LC}}\right) \Rightarrow \text{l'Hospital}$$

$$\left(\lim_{\omega\to\infty} \frac{\omega RC}{1-\omega^2 LC}\right) \underset{=\!=}{\overset{\text{l'Hospital}}{}} \left(\lim_{\omega\to\infty}\frac{RC}{-2\omega LC}\right) \Rightarrow \underline{\underline{0}}$$

e)

siehe Lösung

f)

<u>Tiefpass</u>

a) s. Lösung

b) $$\frac{U_c(s)}{\frac{U_0}{s}} = \frac{\frac{\frac{1}{sC}\cdot R}{\frac{1}{sC}+R}}{R+\frac{\frac{1}{sC}R}{\frac{1}{sC}+R}} = \frac{\frac{R}{1+sCR}}{R+\frac{R}{1+sCR}} \quad \begin{matrix}\cdot(1+sCR)\\ \cdot(1+sCR)\end{matrix}$$

$$\Rightarrow \frac{R}{R+R(1+sCR)} = \frac{\cancel{R}\,1}{1+1+sCR} = \frac{1}{2+sCR} \quad \begin{matrix}:CR\\ :CR\end{matrix}$$

$$\Rightarrow \cancel{\frac{1}{CR}}\cdot\frac{\cancel{1}\ \frac{1}{CR}}{s+\frac{2}{CR}} = \frac{1}{2}\cdot\frac{\frac{2}{CR}}{s+\frac{2}{CR}}$$

$$\underline{U_c(s)} = \frac{U_0}{s}\cdot\frac{1}{2}\cdot\frac{\frac{2}{CR}}{s+\frac{2}{CR}} = \boxed{\frac{U_0}{2}\cdot\frac{a}{s(s+a)}} \quad \text{für } a=\frac{2}{CR}$$

c) Bild-bereich $\bullet\!\!-\!\!\circ$ Zeitbereich

$$\frac{a}{s(s+a)} \bullet\!\!-\!\!\circ\ 1-e^{-at}$$

Somit

$$\boxed{u_c(t) = \frac{U_0}{2}\cdot\left(1-e^{-\frac{2}{CR}\cdot t}\right)}$$

d) s. Lösung