Trigonometrische Funktionen

	0	$\frac{1}{6}\pi$	$\frac{1}{4}\pi$	$\frac{1}{3}\pi$	$\frac{1}{2}\pi$	$\frac{2}{3}\pi$	$\frac{3}{4}\pi$	$\frac{5}{6}\pi$	π	$\frac{7}{6}\pi$	$\frac{5}{4}\pi$	$\frac{4}{3}\pi$	$\frac{3}{2}\pi$	$\frac{5}{3}\pi$	$\frac{7}{4}\pi$	$\frac{11}{6}\pi$	2π
	0^0	30^0	45^0	60^0	90^0	120^0	135^0	150^0	180^0	210^0	225^0	240^0	270^0	300^0	315^0	330^0	360^0
$\sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\tan x$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$\pm\infty$	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$\pm\infty$	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0
$\cot x$	$\pm\infty$	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	$-\frac{\sqrt{3}}{3}$	-1	$-\sqrt{3}$	$\pm\infty$	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	$-\frac{\sqrt{3}}{3}$	-1	$-\sqrt{3}$	$\pm\infty$

Additionstheoreme

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$
$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$
$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$

Doppelter Winkel

$$\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1$$
$$\sin 2x = 2 \sin x \cos x$$
$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$
$$\cot 2x = \frac{\cot^2 x - 1}{2 \cot x}$$

halber Winkel

$$\cos\frac{x}{2} =^{\bullet} \pm\sqrt{\frac{1}{2}(1 + \cos x)}$$
$$\sin\frac{x}{2} =^{\bullet} \pm\sqrt{\frac{1}{2}(1 - \cos x)}$$
$$\tan\frac{x}{2} = \frac{1-\cos x}{\sin x} = \frac{\sin x}{1+\cos x} =^{\bullet} \pm\sqrt{\frac{1-\cos x}{1+\cos x}}$$
$$\cot\frac{x}{2} = \frac{1+\cos x}{\sin x} = \frac{\sin x}{1-\cos x} =^{\bullet} \pm\sqrt{\frac{1+\cos x}{1-\cos x}}$$

Symmetrie

$\cos(-x) = \cos x$	gerade Funktion
$\sin(-x) = -\sin x$	ungerade Funktion
$\tan(-x) = -\tan x$	ungerade Funktion
$\cot(-x) = -\cot x$	ungerade Funktion

$$\boxed{\cos^2 x + \sin^2 x = 1}$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x) \qquad \sin x =^{\bullet} \frac{\tan x}{\pm\sqrt{1+\tan^2 x}}$$
$$\sin^2 x = \frac{1}{2}(1 - \cos 2x) \qquad \cos x =^{\bullet} \frac{1}{\pm\sqrt{1+\tan^2 x}}$$
$$\cos x = \sin\left(\frac{\pi}{2} \pm x\right) \qquad \tan x = \frac{\sin x}{\cos x}$$
$$\sin x = \cos\left(\frac{\pi}{2} - x\right) \qquad \cot x = \frac{\cos x}{\sin x} = \frac{1}{\tan x}$$

$$\sin x + \sin y = 2 \sin\frac{x+y}{2} \cos\frac{x-y}{2}$$
$$\sin x - \sin y = 2 \cos\frac{x+y}{2} \sin\frac{x-y}{2}$$
$$\sin x \cdot \sin y = \frac{1}{2}\big(\cos(x-y) - \cos(x+y)\big)$$
$$\cos x + \cos y = 2 \cos\frac{x+y}{2} \cos\frac{x-y}{2}$$
$$\cos x - \cos y = -2 \sin\frac{x+y}{2} \sin\frac{x-y}{2}$$
$$\cos x \cdot \cos y = \frac{1}{2}\big(\cos(x-y) + \cos(x+y)\big)$$
$$\sin x \cdot \cos y = \frac{1}{2}\big(\sin(x-y) + \sin(x+y)\big)$$

• Vorzeichen je nach Quadranten!

Hyperbelfunktionen

$$\cosh x = \frac{1}{2}(e^x + e^{-x}) \qquad \tanh x = \frac{\sinh x}{\cosh x} = \frac{e^{2x}-1}{e^{2x}+1}$$
$$\sinh x = \frac{1}{2}(e^x - e^{-x}) \qquad \tanh\frac{x}{2} = \frac{e^x-1}{e^x+1}$$

$$\cosh 0 = 1, \quad \sinh 0 = 0, \quad \tanh 0 = 0$$

$$\boxed{\cosh^2 x - \sinh^2 x = 1}$$

$$\cosh(-x) = \cosh x \;,\quad \sinh(-x) = -\sinh x \;,\quad \tanh(-x) = -\tanh x \;,\quad \coth(-x) = -\coth x$$

Additionstheoreme

$$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$$
$$\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$$

$$\cosh\frac{x}{2} = \sqrt{\frac{1}{2}(\cosh x + 1)}$$
$$\sinh\frac{x}{2} = \pm\sqrt{\frac{1}{2}(\cosh x - 1)}, \quad \text{für } \begin{cases} x \geq 0 \\ x < 0 \end{cases}$$

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$
$$\sinh 2x = 2 \sinh x \cosh x$$

$$\operatorname{arsinh} x = \ln\left(x + \sqrt{x^2 + 1}\right)$$
$$\operatorname{arcosh} x = \ln\left(x + \sqrt{x^2 - 1}\right), \quad \text{für } x \geq 1$$